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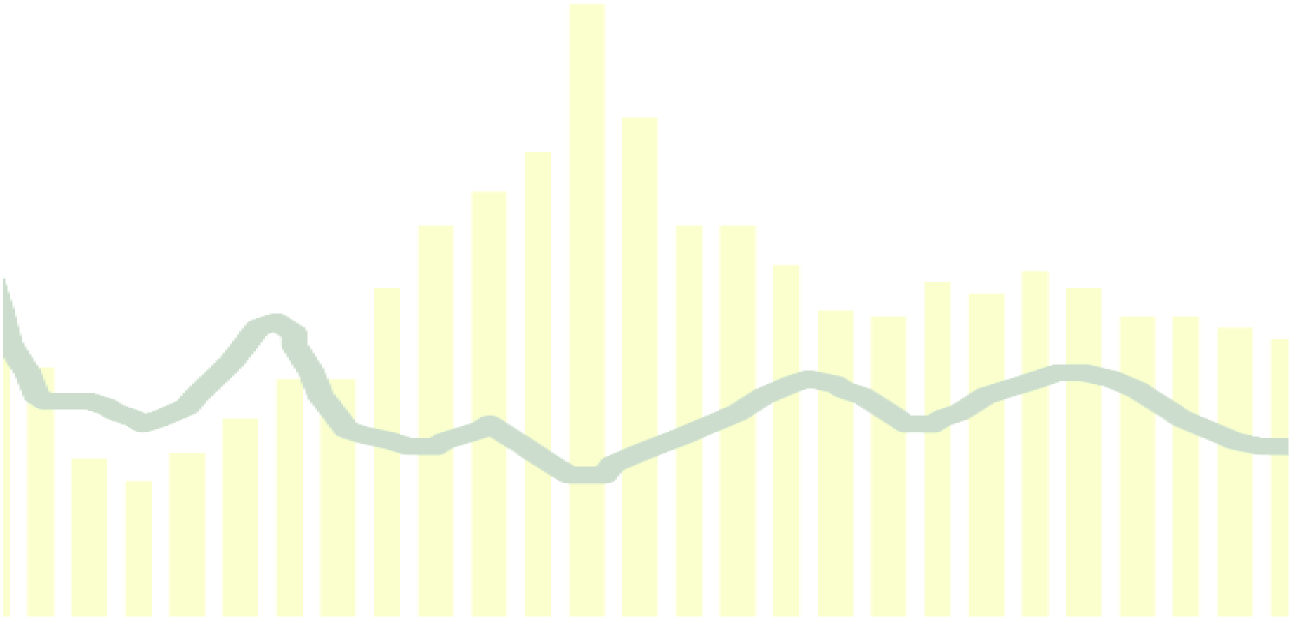
Population Division

Technical Paper
No. 2017/6

Projecting Age-sex-specific Mortality: A Comparison of the Modified Lee-Carter and Pattern of Mortality Decline Methods



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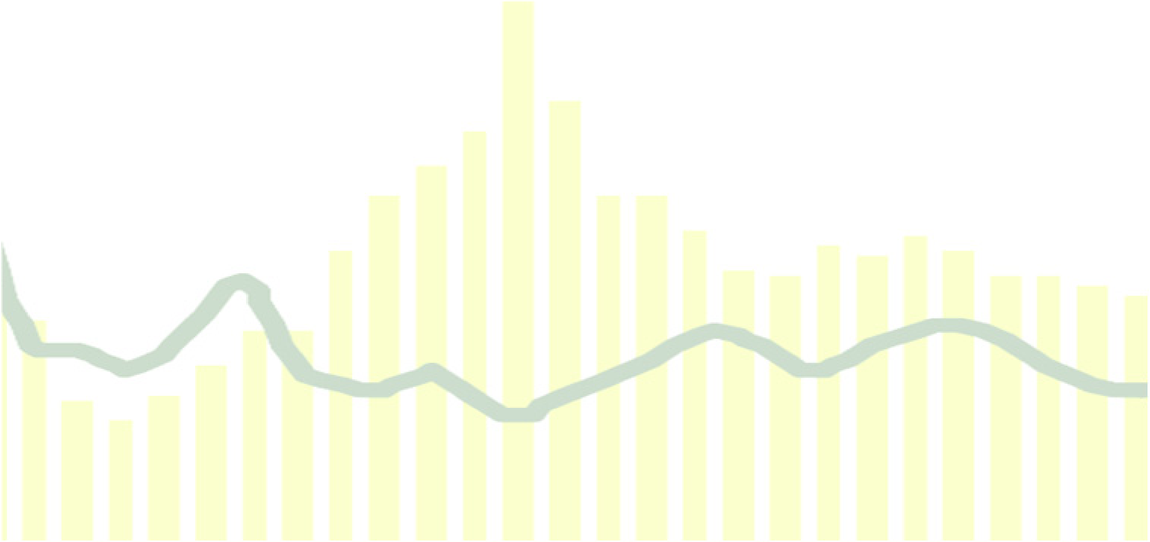
Danan Gu, François Pelletier and Cheryl Sawyer



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PREFACE

This technical paper focuses on the application and comparison of different methods to project the age and sex patterns of mortality within the context of producing the official estimates and projections of the *2017 Revision* of the *World Population Prospects*. We compare the performance of the pattern of mortality decline (PMD) method and the three variants of the modified Lee-Carter (MLC) method as applied to age and sex-specific death rates (m_x) from 1950-1955 to 2010-2015 and used to project m_x from 2015-2020 to 2095-2100 for 155 countries. The paper compares among these different methods, instances where female mortality is projected to be higher than male mortality (sex crossovers) and where death rates between adjacent time periods fluctuate (jumps in $m_{x)}$. Among the three MLC variants, the one using the average age pattern of mortality over the 1950-2015 period produces fewer crossovers but more jumps, whereas the one using the pattern of the last period (2010-2015) with age smoothing produces fewer jumps but more crossovers. Jumps are concentrated in the first few projection periods and in young age groups. Overall, the MLC method regardless of its variants generally worked well for countries with good data quality, whereas the PMD method performed better for countries with lower data quality. This study suggests that the MLC method produces less stable results for future age-sex-specific death rates for countries with relatively low-quality data. It also suggests that a more coherent use of the PMD method may further improve the performance of PMD.

The authors gratefully acknowledge Patrick Gerland and Hana Ševčíková for their technical support and helpful comments during the analytical phase. The authors also acknowledge the insightful inputs of colleagues in the Population Estimates and Projection Section: Lina Bassarsky, Helena Cruz Castanheira, Neena Koshy, Igor Ribeiro, Thomas Spoorenberg and Guangyu Zhang.

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PROJECTING AGE-SEX-SPECIFIC MORTALITY: A COMPARISON OF THE MODIFIED LEE-CARTER AND PATTERN OF MORTALITY DECLINE METHODS

Danan Gu, François Pelletier and Cheryl Sawyer
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1. INTRODUCTION

Projections of life expectancy at birth are needed for many demographic applications but can be a challenge to produce (Bohk and Rau, 2016). In existing literature, several methodological approaches can be found: extrapolation of past trends, expert judgment, and epidemiological models (Booth and Tickle, 2008). Although it is still debatable which among these three produces the most robust forecasts (Bergeron-Boucher et al., 2017; Booth and Tickle, 2008), the extrapolation approach is often considered the most robust because future values of life expectancy are mainly projected from the past empirical estimates with limited subjective judgment (Booth and Tickle 2008; Booth et al. 2006; Oeppen and Vaupel, 2002; Stoeldraijer et al., 2013). The United Nations Population Division currently uses a probabilistic extrapolation approach for projecting life expectancy at birth in its biennial *World Population Prospects* (Castanheira, Pelletier and Ribeiro, 2017; Raftery et al., 2013; Raftery, Lalic and Gerland, 2014; United Nations, 2017a).

The projection of age-specific death rates is an additional challenge. In the projection of age-specific death rates, it is necessary not only to take into consideration the possible change in the pattern of mortality improvement, but also to avoid possible sex crossovers in death rates and jumps in death rates between adjacent time periods (Bergeron-Boucher et al., 2017; Bohk and Rau, 2016; Lee and Carter, 1992; Li and Lee, 2005; Ševčíková et al., 2016).

The Lee-Carter (LC) model is one widely used extrapolation method for projecting age-specific mortality. The LC model assumes that the logarithms of age-specific death rates follow a linear trend with fixed improvement rate (Lee and Carter 1992). The LC model works well in many cases, especially when the projection horizon is a half century or less (Bergeron-Boucher et al., 2017; Lee and Miller, 2001; Tuljapurkar et al., 2000). However, growing evidence has shown that such an assumption of fixed improvement rates may not be correct, especially for high ages and when the life expectancy at birth reaches a high level (Bergeron-Boucher et al., 2017; Booth, Maindonald, and Smith, 2002; Booth and Tickle 2008; Kannisto et al. 1994). This is because future improvements in mortality are expected to entail a shift of the improvements from younger ages to older ages (Li and Lee, 2005), and constant improvement rates would not be consistent with future trajectories of mortality. Recognizing the limitation of the conventional LC model, many researchers have proposed modifications to the LC model that incorporate the mortality improvement shift in the projection (Li and Lee, 2015; Li et al., 2013; Bohk and Rau, 2016). Furthermore, because of possible sex crossovers in death rates at some ages in the projection model when life expectancy at birth reaches high levels, some researchers have proposed coherent approaches assuming the same age pattern of mortality improvement for both sexes instead of the extrapolation of sex-specific trends (Bohk and Rau, 2016; Carins et al., 2011; Hyndman et al., 2013; Li and Lee, 2005) to avoid implausible sex crossovers in death rates.

Another extrapolation method is the pattern of mortality decline (PMD) method. The PMD method was developed by analysts of the United Nations Population Division (Andreev, Gu, and Gerland, 2013) and has been implemented in the last four revisions (2010, 2012, 2015, and 2017) of the *World Population Prospects*. The central idea of PMD is to project relative reductions in mortality for each age group over the projection horizon. The age pattern of relative reduction is governed by the level of life expectancy at birth. These relative reductions can be transformed into death rates with a corresponding life expectancy, and both the sex coherent pattern and the shift pattern can be considered in the PMD method.

More recently, Bergeron-Boucher et al. (2017) applied a compositional data analysis (CoDa) approach to project life table deaths instead of death rates. This approach models and forecasts a redistribution of life table deaths for a given sex in a specified year. A rotated shift of deaths from young ages to older ages and a coherent mortality pattern can be applied if desired. According to the authors, the results of the CoDa method are less biased than the LC method based on empirical data for females in 15 Western European countries (Bergeron-Boucher et al., 2017). The underlying mechanism of the CoDa approach is similar to PMD.

All of these methodological developments in the area of mortality projections have improved our knowledge about how to better project mortality and about the strength and weakness of the different methods. Yet, for the most part, existing methodological comparisons are based on countries with good data (e.g., Li and Lee, 2005; Li, Lee, and Gerland, 2013; Bergeron-Boucher et al., 2017). It is unclear which method would be more appropriate for countries with mortality data of poorer quality. However, in producing the projections of the *World Population Prospects*, mortality patterns had to be projected for over 200 countries with varying quality of mortality estimates by age and sex. In such a context, a comparison of different methods has a far-reaching significance for adopting or further refining appropriate methods for countries with different degrees of data quality. The purpose of this study is to compare the performance of several variants of the modified LC (MLC) and PMD methods in terms of sex crossovers and jumps in projected m_x. The results provide a rationale for the choice of method to project age-specific death rates for different groups of countries in the *2017 Revision* of the *World Population Prospects*.

Before elaborating on the different methods to project age-specific mortality rates, it should be noted that within the *World Population Prospects*, life expectancy trends are projected before the age patterns of mortality. In the first step, female life expectancies at birth for each country are projected using a Bayesian hierarchical model (Raftery et al., 2013). In the second step, the gap between female and male life expectancy at birth is projected in order to derive the corresponding male life expectancy (Raftery, Lalic and Gerland, 2014). Both procedures were updated and revised during the preparation of the *2017 Revision* (Castanheira, Pelletier and Ribeiro, 2017; United Nations, 2017a).

2. METHODS

2.1. Pattern of mortality decline (PMD)

The PMD method assumes that the future decline in age-sex-specific mortality will follow a certain pattern ρ_x with the increase in life expectancy at birth. Denote $m_x(t_1)$ and $m_x(t_2)$ the age-specific death rate at age *x* at time t_1 and t_2, corresponding to life expectancy at birth $e_0(t_1)$ and $e_0(t_2)$, respectively. It follows:

$$\ln m_x(t_2) = \ln m_x(t_1) - k(t_{12})\rho_x(t_{12}), \qquad (1)$$

where, $\rho_x(t_{12})$ is the age-specific pattern of mortality decline from time t_1 to time t_2 $(\sum_x \rho_x(t_{12}) = 1)$, $k(t_{12})$ is a parameter governing the level of mortality decline over the period. The age-sex-specific values of ρ_x for e_0 up to 75-80 years for males and for e_0 up to 80-85 years for females were derived from empirical data of all countries in the Human Mortality Database (HMD) (Andrew, Gu and Gerland, 2013)[1]. The term ρ_x refers to the median of declines in all countries in HMD for a given age *x* and for a given level of e_0. In PMD, the value of ρ_x is smoothed across ages with a cubic spline approach. The level of e_0 is considered in five-year intervals. In other words, ρ_x would be the same for any e_0 in the five-year interval, for example (75, 80). The estimated ρ_x values for e_0 greater than the maximum level in HMD (i.e., greater than 75-80 years for males and greater than 80-85 years for females) were extrapolated based on fitted ordinary least square regressions within each age group with a slight smoothing over ages. In the current version of PMD, the fitted ρ_x values are projected for e_0 up to 105-110 years for both males and females (Andreev, Gu, and Gerland, 2013). As e_0 rises, there is a shift in ρ_x values from younger ages to older ages, indicating that when e_0 reaches a high level, additional gains in life expectancy are increasingly due to a reduction in mortality at older ages rather than at younger ages. The higher the life expectancy, the greater the reduction in mortality at older ages (Andreev, Gu, and Gerland, 2013). $k(t_{12})$ is then estimated by iteration to match e_0 with a very small tolerance (e.g., <0.001). In the *2017 Revision*, the ρ_x values are sex-specific (Andreev, Gu, and Gerland, 2013). To improve stability and cross-national consistency, a blended pattern of age-sex mortality rates was applied by using the weighted average of ρ_x and the improvement pattern derived from an extended model life table (United Nations, 2017b); the weight of ρ_x decreases from 1 in the first period (2015-2020) to 0.5 in 2095-2100. In some cases, sex crossovers in m_x were adjusted by providing an extrapolated sex ratio for the

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[1] The Human Mortality Database is a collection of mortality estimates for 38 countries. The data have been extensively evaluated and corrected before inclusion in the database, and thus provide a sound basis for estimating mortality models (. The HMD may be accessed at www.mortality.org.

projection period based on either the trend analysis or other methods. We label the results of the PMD approach without using such an adjustment for sex crossovers as “PMD-unadj”.

In the *2017 Revision*, the PMD method was implemented for 130 countries that had 90,000 inhabitants or more on 1st July 2017, and for 86 of these an adjustment was applied for sex crossovers in m_x. We have labelled the final outcome of the PMD from the *2017 Revision* as “PMD-WPP” in tables 1 and 2.

2.2. Lee-Carter method

2.2.1. Lee-Carter model and its variants

The standard Lee-Carter model (Lee and Carter, 1992) can be expressed as

$$\ln m_x(t) = a_x + b_x k(t) + \varepsilon_x(t)\,, \quad \varepsilon_x(t) \sim N(0, \sigma_\varepsilon^2)\,, \qquad (2)$$

where, $\ln m_x(t)$ is natural logarithm of the death rate $m_x(t)$ at age *x* at time *t* (a year or a five-year period). The parameter a_x is the baseline age pattern of mortality, which is normally constant over time; b_x is the average rate of change in age-specific death rate for a unit change in parameter $k(t)$, $\sum_x b_x = 1$; the parameter $k(t)$, the regression of coefficient of b_x, is an index of the overall level of mortality at time *t,* and $\varepsilon_x(t)$ is the residual at age *x* and time *t* (Ševčíková et al., 2016). In calculation, a_x is usually the average of $\ln m_x(t)$ for the empirical data. In the case of the *2017 Revision*, it is the average of $\ln m_x(t)$ over the period from 1950-1955 to 2010-2015. $k(t)$ is identified by setting $\sum_{t=1}^{T} k(t) = 0$, where *T* is the number of past time periods for which data are available. In the case of the *2017 Revision*, *T* equals 13.

Li and colleagues (Li and Lee, 2005; Li, Lee, and Gerland, 2013) proposed a modified Lee-Carter (MLC) model that uses the same age pattern for mortality improvement rate (b_x) for males and females or for subpopulations to ensure coherent changes in mortality age pattern. Their modified LC method also accommodates a shift in age pattern of mortality improvement from younger ages to older ages to address the criticism for the assumption of constant b_x in the standard LC model. They assume that the shift starts after life expectancy at birth for both sexes combined reaches 80. The shifted age pattern (B_x) is the weighted average between the age pattern of improvement from the standard LC method (b_x) and an ultimate age pattern (b_x^u) governed by the level of projected life expectancy at birth. b_x^u is obtained by assuming a proportional increase in b_x^u for ages 70 or older (normally assuming $b_{70}^u = b_{65}^u$ and proportionally increasing the rate at other older ages) and an identical value of b_x^u for ages below age 65 with a constraint that the sum of b_x^u at all ages equals 1. For k_t, Li and colleagues suggest forecasting it with a random walk without drift or with an autoregressive model (AR) with intercept for each sex. They argue that such a modification could avoid possible sex crossovers in death rates and also assure that the mortality rate at age 0 would not fall below the mortality rate at age 15-19, an age group with relatively low mortality.

2.2.2. Lee-Carter model as applied in the World Population Prospects

In the *2017 Revision*, the modified LC (MLC) method proposed by Li and colleagues (2005, 2013) was implemented for 25 countries with high-quality data, that is, with data that permit valid extrapolation of the country's m_x trends without using a global model. The estimation period included thirteen five-year periods from 1950-1955 to 2010-2015. The projection horizon covers the period from 2015-2020 to 2095-2100. However, there is one crucial difference between the original MLC proposed by Li and colleagues and the one applied in the *2017 Revision*; that is, in the former, the projected $k(t)$was used to obtain life expectancy at birth, whereas in the *2017 Revision* $k(t)$ was determined by the projected life expectancy at birth that had been derived from Bayesian hierarchical models, as described above in section 1. Given the predetermined life expectancy projections for males and females, $k(t)$ was iteratively estimated with a small tolerance to ensure that the life expectancy derived from age-specific death rate matched the given life expectancy. When the life expectancy at birth exceeded 80 years, the shifted age pattern of mortality was applied (Li, Lee, and Gerland, 2013). In other words, the implementation of the MLC method in the *2017 Revision* was similar to that of the PMD method in that it was scaled to the previously determined level of life expectancy (United Nations, 2017a).

Three variants of the MLC method were tested in the *2017 Revision*, using different baseline age patterns of mortality: the age pattern of the last estimation period without age smoothing (MLC1), the pattern of the last period with age smoothing (MLC2), and the average age pattern over the entire estimation period from 1950-1955 to 2010-2015 (MLC3).

2.3. Occurrence rates of sex crossovers and jumps in age-specific death rate

A **sex crossover** of mortality is defined when females have a higher death rate than males at age x (x=0, 1-4, 5-9, 10-14, ..., 95-99, 100+) in a projection period t in a given population. The occurrence rate of crossover is the proportion of crossovers among the total number of m_x under study. The occurrence rate of crossovers can be calculated by age, period or other characteristics such as levels of life expectancy.

A **jump** in mortality is defined when m_x in projection period $t+1$ is higher than that in period t for a given sex in a given population. The occurrence rate of jumps is the proportion of jumps among the total number of m_x increments under study. The occurrence rate of jumps can be calculated by age, sex, or other characteristics.

2.4. Country classifications

The total number of countries in this study is 155, which refers to countries with a total of 90,000 inhabitants or more in 2017 for which the PMD or MLC method was applied for mortality projection in the *2017 Revision*. Countries where the age pattern of mortality was estimated using model life tables or explicit modelling of the impact of HIV/AIDS on mortality are excluded from this analysis (United Nations, 2017a). These 155 countries were first classified according to their inclusion in the Human Mortality Database (HMD), which requires relatively high standards of data quality (Barbieri et al., 2015). Only 38 of the 155 countries have been included in this database. Even among the HMD countries, there are countries or data for selected years with a lower quality, especially at older ages or further in the past. Of the 38 HMD countries, 25 were considered to have higher data quality, consistent with research findings in the literature (e.g., Jdanov et al., 2008; Li, Lee, and Gerland, 2013; Thatcher, Kannisto, and Vaupel, 1998). These 25 countries are labelled HMDa in the present analysis. The 13 HMD countries with lower-quality data, categorized here as HMDb, are mainly in Eastern Europe. The remaining 117 countries are classified as non-HMD countries. For some analyses, the HMDa countries are further broken down according to the three MLC variants. Appendix table A1 lists the countries included in each group.

2.5. Variants of projection methods

The results from each of the four projection methods (PMD-unadj, MLC1, MLC2, and MLC3) for 155 countries are compared in terms of occurrence rates of sex crossovers in m_x between males and females and jumps in m_x between two adjacent periods, to examine the advantages and disadvantages of each method. In addition to these comparisons, the results of the *2017 Revision* are also presented according to the method chosen for each country.

For all categorizations of different factors under study, we used the series 0, 1-4, 5-9, 10-14, ..., 95-99, 100+ for 22 age categories and the series 2015-2020, 2020-2025, ..., 2095-2100 for 17 five-year projection periods.

3. RESULTS

3.1. Sex crossovers in age-specific death rate

3.1.1. General assessment of crossovers

Figure 1 presents the proportion of countries having at least one sex crossover in the projection period by method of projection for different groups of countries. Overall, the proportion of countries having at least one sex crossover is high for nearly all combinations of methods and groups of countries. However, the average occurrence rate of crossover for age-specific m_x in a given time period is not nearly as pronounced.

The upper panel of table 1 presents the average overall occurrence rate of sex crossovers in m_x for different groups of countries in the projection periods, from 2015-2020 to 2095-2100, for each of the four approaches. Overall, MLC3 performed the best (1.5 per cent), whereas PMD-unadj performed the worst (6.8 per cent). In the case of HMDa countries, all three MLC approaches produced a lower occurrence rate of crossovers than PMD-unadj. The PMD-unadj method scored the lowest occurrence rate of sex crossovers in non-HMD countries (6.2 per cent) compared with HMD countries as a whole (8.5 per cent) and HMDa countries (7.5 per cent). When comparing HMD and non-HMD countries, the crossover rate is smaller in non-HMD countries except for the MLC3 method.

The lower panel of table 1 presents the occurrence rates of sex crossovers in the *2017 Revision* for groups of countries according to the projection method implemented. Among the 130 countries for which PMD was used in the *2017 Revision*, an adjustment was applied for the sex ratio of m_x in 86 countries. The occurrence rate of crossovers for the 130 PMD countries is 3.2 per cent (PMD-WPP) and it would be 6.7 per cent if no adjustment had been applied to these 86 countries. For the 25 MLC countries, the choice between the three MLC variants was conditioned on the minimization of crossovers and jumps. In the cases of MLC2 and MLC3, these two variants minimized crossovers for the countries for which they were employed (bold numbers in table 1). However, in the case of MLC1, the method did not produce the lowest occurrence rate of crossovers. The MLC1 approach was applied in these countries because it yielded fewer number of jumps in m_x compared to MLC2 and MLC3, as will be shown in section 3.2.

Figure 1. Proportion of countries having at least one sex crossover by projection method and country group

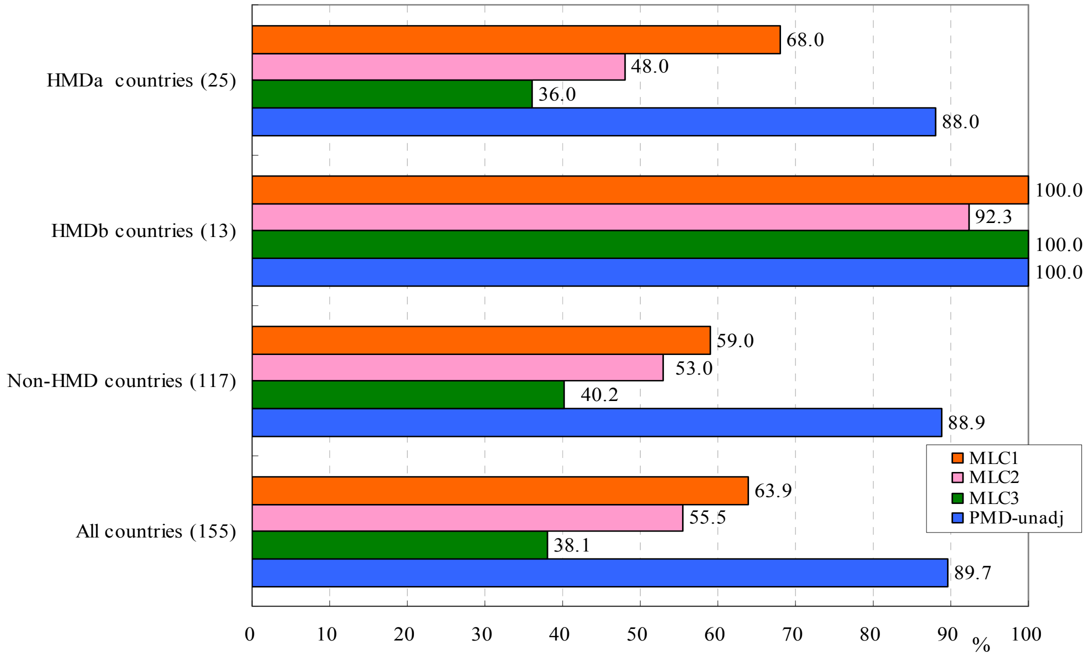


TABLE 1. AVERAGE OCCURRENCE RATES (PER CENT) OF SEX CROSSOVERS IN M_X IN PROJECTION PERIODS FROM 2015-2020 TO 2095-2100 FOR DIFFERENT PROJECTION METHODS, BY COUNTRY GROUP

	MLC 1	MLC2	MLC3	PMD-unadj	PMD-WPP
	(1)	(2)	(3)	(4)	(5)
Country classification by quality of data					
All countries (155)	3.9	3.4	1.5	6.8	---
All HMD countries (38)	5.3	4.6	1.0	8.5	---
HMDa countries (25)	2.2	1.4	1.2	7.5	---
HMDb countries (13)	11.2	10.6	0.7	10.6	---
Non-HMD countries (117)	3.5	3.0	1.7	6.2	---
Country classification according to the approach used in the *2017 Revision*					
PMD-WPP (130)	4.3	3.7	1.6	6.7	**3.2**
MLC1 (8)	**1.4**	1.2	1.4	4.4	---
MLC2 (5)	1.2	**0.3**	2.8	7.2	---
MLC3 (12)	3.2	2.0	**0.4**	9.6	---

NOTES: (1) The occurrence rate for a country is calculated from the number of occurrences of sex crossovers over a total number of m_x projections for 22 age groups and 17 five-year periods. The rate in the table is the average of all countries in a given country group with the listed approach. (2) Of 130 countries projected with PMD in the *2017 Revision* (PMD-WPP), 86 countries had an adjustment applied for sex ratio of m_x.

3.1.2. Occurrence rates of sex crossovers by age group, female-male gap in e_0*, and period*

Figure 2 shows the occurrence rate of sex crossovers in m_x for each method and country group according to several factors. The upper panel (A) of figure 2 shows that the occurrence rate of sex crossovers in m_x varies considerably by age group. The sex crossovers are mainly concentrated at young ages, especially occurring in age group 1-4 and at age 0. The sex crossover rates for HMDb countries at young ages are particularly high, except with the MLC3 method.

When the gap between male and female life expectancy is narrow, we can expect that sex crossovers for specific age groups would be more likely (because the male and female m_x are closer together to start with). The middle panel (B) of figure 2 confirms that there is a general tendency for a smaller female-male gap in e_0 to produce a higher rate of sex crossovers. For HMDa countries, the lower occurrence rate when the female-male e_0 gap is less than 2 years is due to a very small number of occurrences (only 3 crossover incidences in total in 2040 for Iceland; no other HMDa countries had such a small gap).

The bottom panel (C) of figure 2 reveals discrepant sex crossover patterns over time across the three country groups. In non-HMD countries, the occurrence rate over the entire projection period (2015-2100) produced by each of the three MLC methods is more or less the same: the rate slightly decreases from 3-4 per cent in 2015-2020 to 2-3 per cent in 2030-2050 and slightly increases back to the starting levels in 2075-2100; the occurrence rate for PMD-unadj is similar to those of MLC methods before 2050, but it increases to 10 per cent in 2075-2100.

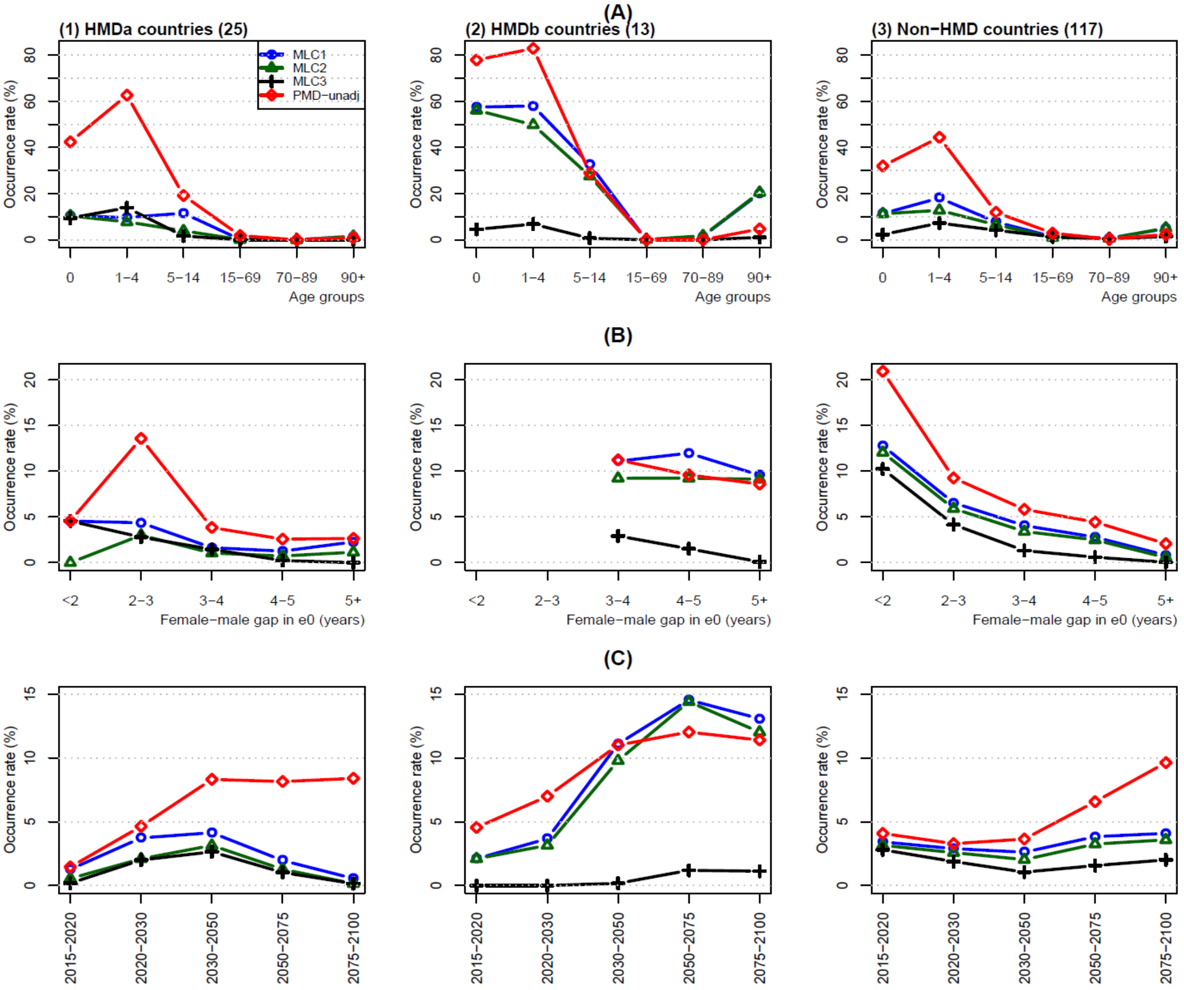
In the case of HMDb countries, the rate of sex crossovers produced by MLC3 is very low over the projection period; the rates of sex crossovers produced by MLC1 and MLC2 increase from 2-3 per cent in the first period to nearly 15 per cent in 2050-2075 and then decline slightly to 12-13 per cent in 2075-2100. The time pattern of crossovers produced by PMD-unadj is similar to those of MLC1 and MLC2, while the rate is higher before 2050 and slightly lower after 2050 for PMD-unadj.

For HMDa countries the occurrence rate produced by the three MLC variants increases slightly from 1-2 per cent in 2015-2020 to 3-4 per cent in 2030-2050 and diminishes thereafter, while the occurrence rate produced by PMD-unadj increases from 2 per cent in 2015-2010 to 8 per cent in 2030-2050 and remains steady afterwards.

Multivariable analysis of crossovers that simultaneously models age group, period, and the female-male gap of life expectancy at birth, reaches the same conclusions mentioned above (appendix table A2), indicating that those findings are relatively robust. However, the multivariable regression models reveal that the sex crossover rate produced by MLC3 diminishes over time, while the rate produced by other three methods tends to be flat or increase over time, especially in the case of PMD. Although the sex crossover rate produced by MLC3 is lower in HMDa countries compared to non-HMD countries, the difference between these two groups of countries is not statistically significant. The lower rate for HMDa countries than for HMDb countries produced by MLC3 is also not significant in multivariable analysis.

Overall, PMD-unadj produces the highest sex crossover rate among the four approaches and MLC3 produces the lowest rate. All methods except MLC3 produce higher sex crossover rates for HMDb countries than for non-HMD or HMDa countries.

Figure 2. Average occurrence rate of sex crossovers in m_x from 2015 to 2100 by (A) age group, (B) gap between female and male life expectancy at birth, and (C) period, by projection method and country group

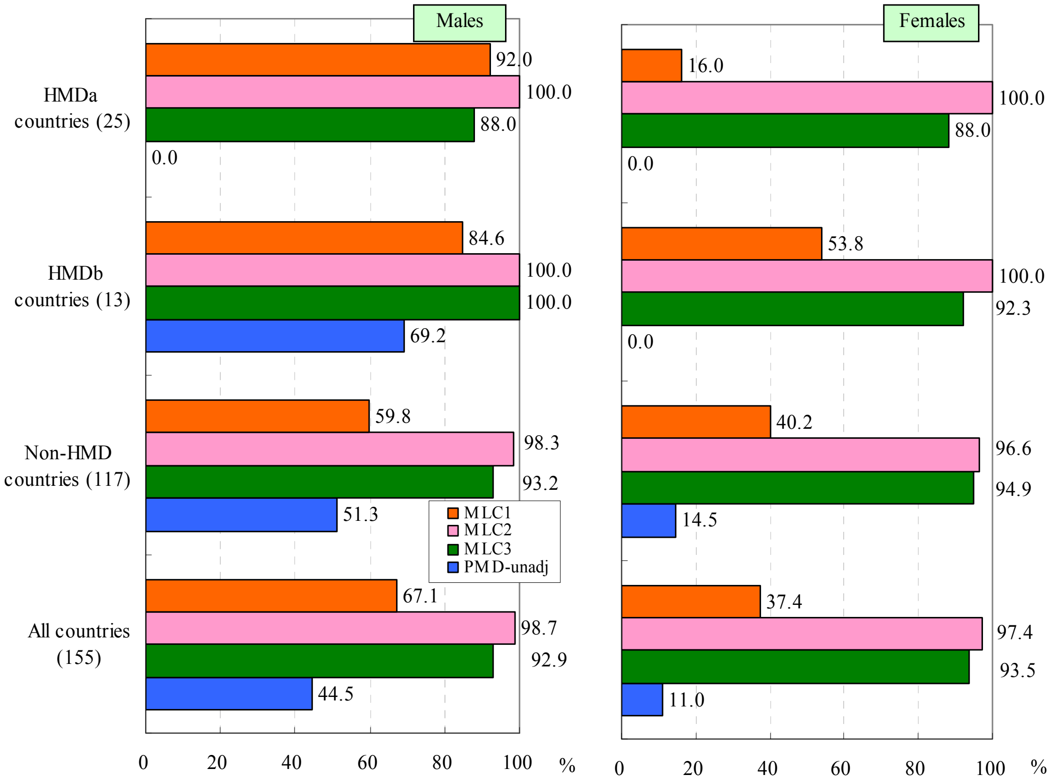


3.2. Jumps in age-specific death rate between adjacent periods

3.2.1. General assessment of jumps

Figure 3 presents the proportion of countries having at least one jump in m_x by projection method and sex for the three country groups. For males, the occurrence of any jump is quite high for all methods in all country groups, with the exception of PMD-unadj for HMDa countries. For females, the PMD-unadj method produces a relatively lower proportion having any jump; by contrast, the MLC3 and MLC2 produce a relatively higher occurrence of any jump. While the PMD-unadj produces no jumps for both males and females in HMDa countries and for females in HMDb countries, the number of countries having at least one jump produced by the three MLC methods is quite high. About 90 per cent or more of the countries have jumps when MLC2 and MLC3 are used.

Figure 3. Proportion of countries having at least one jump in m_x by sex, projection method and country group



However, the high proportion of countries with at least one jump does not mean that these jumps occur in a high proportion of age groups or periods. In fact, table 2 shows that the occurrence rates of jumps in m_x between two adjacent periods of a projection are much lower than the occurrence rates of sex crossovers that were shown in table 1. On average, the occurrence rate of jumps is lower for the PMD-unadj method compared to the three MLC methods. Among the three MLC methods, MLC3 produces the highest occurrence rate of jumps, while MLC1 has the lowest occurrence rate of jumps. In terms of the occurrence rate of jumps by country group, HMDa countries have the lowest rate regardless of the method applied, while HMDb countries are associated with the highest rate when any one of three MLC approaches is employed. The rates for non-HMD countries for the three MLC variants, but are higher than for the HMD groups when the PMD-unadj approach is applied. There is a distinctive gender pattern across the four methods. The three MLC methods yield similar patterns for males and female with the MLC3 producing the highest rate and MLC1 the lowest rate for all country groups, yet the PMD-unadj produces a rate higher than MLC1 for males, and lower than MLC1 for females.

The bold numbers in the lower panel of table 2 are the occurrence rates for jumps in the *2017 Revision* according to the actual method implemented. In the *2017 Revision*, 86 PMD countries had an adjustment applied for sex crossovers in m_x, while no adjustment was applied for jumps. This increases slightly the occurrence rate of jumps for both males and females in the PMD countries—from 1.1 percent (PMD-unadj) to 1.2 per cent (PMD-WPP) for males, and from 0.2 to 0.3 per cent for females.

Table 2. Average occurrence rate (per cent) of jumps in M_X in projection periods from 2015-2020 to 2095-2100 for different projection methods, by country group

	MLC 1	MLC2	MLC3	PMD-unadj	PMD-WPP
	(1)	(2)	(3)	(4)	
Country classification for HMD vs. non-HMD					
Males					
All countries (155)	0.7	1.2	2.5	1.0	---
All HMD countries (38)	1.5	2.0	3.1	0.3	---
HMDa countries (25)	0.2	0.7	0.8	0.0	---
HMDb countries (13)	4.0	4.6	7.3	0.9	---
Non-HMD countries (117)	0.5	0.9	2.4	1.2	---
Females					
All countries (155)	0.6	1.1	3.2	0.2	---
All HMD countries (38)	0.8	1.3	3.3	0.0	---
HMDa countries (25)	0.0	1.0	3.1	0.0	---
HMDb countries (13)	2.4	2.7	8.3	0.0	---
Non-HMD countries (117)	0.6	1.0	3.1	0.3	---
Country classification according to the approach used in the *2017 Revision*					
Males					
PMD-WPP (130)	0.8	1.3	2.9	1.1	**1.2**
MLC1 (8)	**0.2**	0.6	1.0	0.0	---
MLC2 (5)	0.3	**0.7**	0.8	0.0	---
MLC3 (12)	0.2	0.6	**0.7**	0.0	---
Females					
PMD-WPP (130)	0.7	1.2	3.6	0.2	**0.3**
MLC1 (8)	**0.0**	0.5	1.0	0.0	---
MLC2 (5)	0.0	**0.8**	0.5	0.0	---
MLC3 (12)	0.1	0.6	**0.6**	0.0	---

Note: (1) The occurrence rate of a country is calculated by the number of occurrences of jumps in m_x over a total number of m_x projections for 22 age groups and 17 five-year periods. The rate for each group is the average of all countries in the given group with the given approach. (2) Of 130 countries projected with PMD in the *2017 Revision* (PMD-WPP), 86 countries had an adjustment for sex ratio in m_x (jumps were not adjusted for in PMD -WPP).

3.2.2. Jumps by age group, life expectancy, and period

The upper panel (A) of figure 4 reports the patterns of jumps in m_x by age for males. It shows that the jumps are mainly concentrated at young ages, especially age group 1-4. With the exception of ages 90+ in non-HMD and HMDb countries, the MLC3 method yields the highest occurrence rate of jumps across all ages compared to the other three methods. For HMDa countries, the occurrence rate of jumps in most cases is close to zero, except for age group 1-4 when the MLC2 method is applied. For HMDb countries, the jump rate is high for all three MLC methods for ages below 15. For non-HMD countries, the jump rate lies between the two HMD groups, except the rate for ages 90+ produced by PMD-unadj, which is higher than for the HMD groups.

The middle panel (B) of figure 4 illustrates the occurrence rate of jumps in m_x for males by life expectancy at birth. Over all levels of life expectancy, PMD-unadj produces the lowest rate of jumps for HMD countries, whereas MLC3 produces the highest rate for all three groups of countries. The rate produced by PMD-unadj has no association with the level of life expectancy for all groups of countries, whereas the rate produced by all three MLC variants seems to be associated with the level of life expectancy for HMDb countries. The MLC3 method produces higher occurrence rates of jumps when e_0 levels are between 85-90 years. For HMDa countries, almost no jumps are produced across all four methods. For HMDb countries, all three MLC variants produce a higher rate of jumps than PMD-unadj at levels of life expectancy less than 70 years or 75-85 years.

The bottom panel (C) of figure 4 reveals that most jumps occur in the first period of the projection (i.e., 2015-2020) for all three groups of countries. For HMDb countries, the jump rates produced by the three MLC methods tend to increase over time after 2050.

Figure 5 reveals that females, in general, have similar patterns to males with respect to age groups, periods, and levels of life expectancy, with a few exceptions. For example, when the three MLC methods are applied to HMDb countries, females have a lower rate of jumps than males, with the exception of the rate over all age groups when MLC3 is applied (middle chart of panel A). In the analysis by level of life expectancy (panel B), females have a similar pattern to males, but for the HMDb countries the pattern is shifted to the next higher level of life expectancy when the three MLC methods are applied. In the non-HMD countries when MLC3 is applied, the highest rate of jumps is found when the life expectancy equals to 85-90 year in males and equals to 80-85 years in females.

Multivariable logistic regression models (see appendix table A3) present a more holistic view with age groups, periods, and levels of life expectancy simultaneously taken into account. The multivariable analysis shows that, on average, MLC3 produces more jumps than the other three methods for both males and females. PMD-unadj performs better than the three MLC methods in producing fewer jumps in females, although PMD-unadj produces a higher jump rate than MLC1 in males. In the case of life expectancy, the level of 85-90 years is associated with higher odds of jumps than other levels for males, whereas it is the level age group of 80-85 years that is associated the highest odds of having a jump for females. HMDa countries have the lowest rate of jumps while HMDb countries have the highest rate using any of four methods. These multivariable results confirm the conclusions from the univariate analysis in table 2 and figures 4 and 5.

Figure 4. Average occurrence rate of jumps in m_x between two adjacent periods for males by age group, life expectancy at birth, projection period, projection method and country group

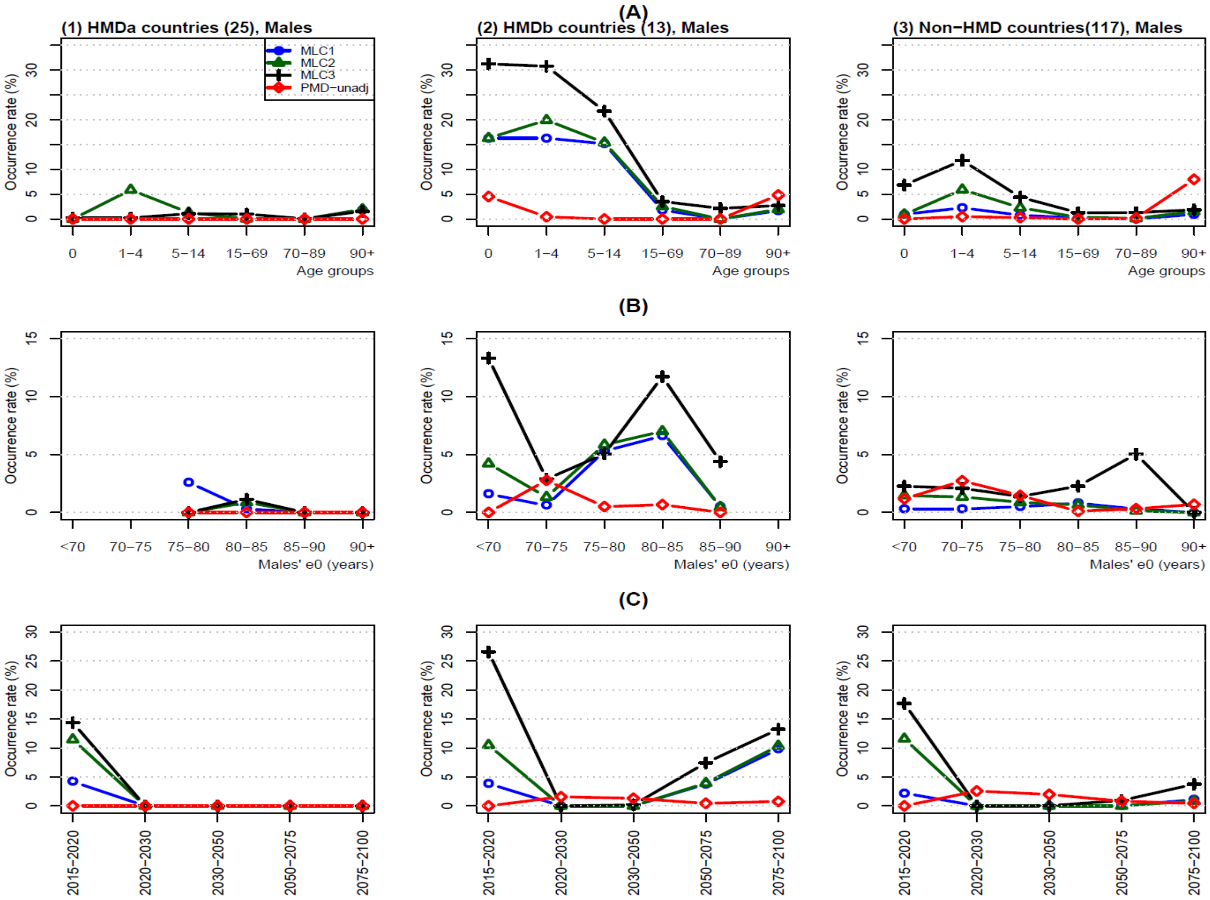
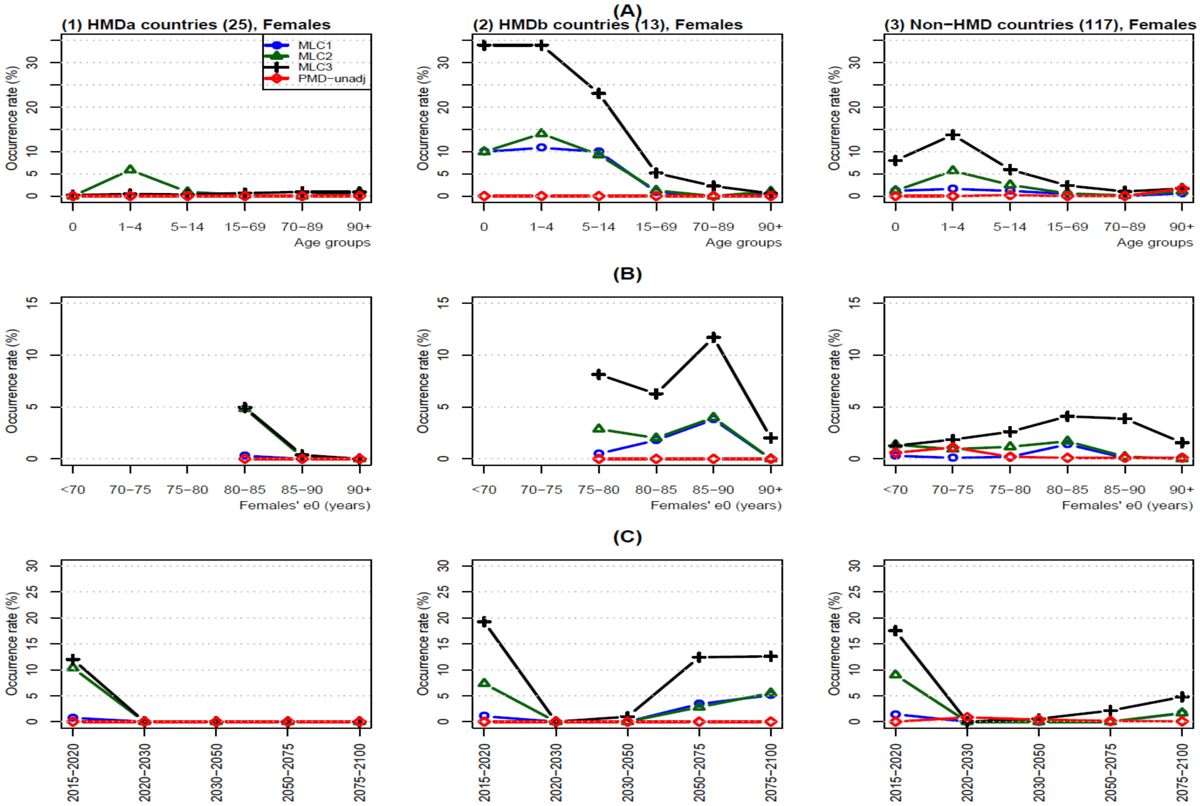


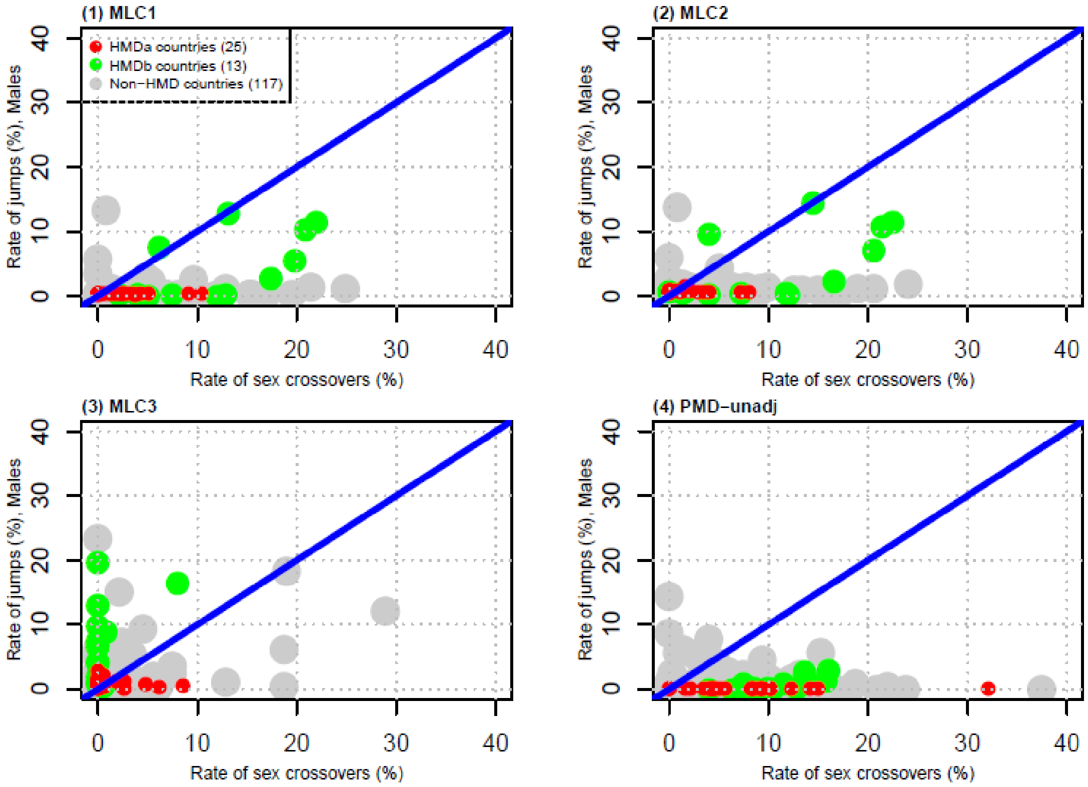
Figure 5. Average occurrence rate of jumps in m_x between two adjacent periods for females, by age group, life expectancy at birth, projection period, projection method and country group



3.3. A summary assessment for sex crossovers and jumps in age-specific death rate

We compare the sex crossover rate against the rate of jumps in m_x of each country to gain a better understanding of the strengths and weaknesses of each projection method. Figure 6 illustrates such a comparison for males. Overall, the results suggest that MLC approaches are not appropriate for HMDb countries, while it is better to apply MLC for HMDa countries. In the cases of MLC, MLC1 provides very small numbers of jumps in m_x for all HMDa countries. However, MLC1 produces higher numbers of sex crossovers for most HMDa countries (also see table 1 and figure 1). Thus, overall, MLC1 is not the most appropriate approach for all HMDa countries. With both sex crossovers and jumps in m_x being taken into consideration, the *2017 Revision* applied MLC1 to eight of the HMDa countries, MLC2 to five countries in the group, and MLC3 to the remaining 12 countries. For HMDb and non-HMD countries, the MLC methods were considered not adequate overall.

Figure 6. Comparisons between occurrence rate of sex crossovers and males' occurrence rate of jumps in m_x by projection method and country group



4. CONCLUSION

This study compares different projection methods for age-sex-specific death rates for 155 countries that are classified into three groups according to the quality of available data. Crossovers between sexes and jumps in the death rate m_x between adjacent periods of the projections are used to compare the projection methods. The study finds that among the three modified Lee-Carter (MLC) variants, the variant based on the average age pattern of the estimation periods (MLC3) produces fewer sex crossovers but more jumps in m_x, whereas the variant using the age pattern of the last estimation period without smoothing (MLC1) produces fewer jumps in m_x but more sex crossovers. Jumps are mainly concentrated in the first few projection periods, and both sex crossovers and jumps in m_x are largely found at very young ages. The unadjusted Pattern of Mortality Decline (PMD-unadj) method produces relatively few jumps in m_x, but a relatively high level of sex crossovers.

Several of these results have straightforward interpretations. MLC3 has fewer crossovers than other methods because it uses the sex-specific average age pattern of mortality over the entire period from 1950-1955 to 2010-2015, and assumes that both sexes will have the same age-specific improvement pattern of mortality in the future. However, when the age-specific death rate in the last estimation period does not follow the average pattern of the entire estimation period—which is the case for many countries—the MLC3 approach will cause some jumps in m_x between the last estimation period (2010-2015) and the first projection period (2015-2020). This is why most jumps occur in the first projection period.

For MLC1, the opposite is the case. Because MLC1 applies the age pattern of mortality of the last estimation period to the projection period, and assumes that both males and females will have the same age pattern of mortality change in the future, it is thus expected to have few jumps between two adjacent periods in the future. However, when the age patterns of death rates of males and females in the last estimation period do not follow their past trajectories, as it is the case in some countries, it is possible that males' death rate at a given age in the projection can fall below that of females of the same age; especially when the age-specific rates are derived from a pre-projected life expectancy at birth and when the female-male gap in life expectancy is narrow. For MLC2, the results are between MLC1 and MLC3 since MLC2 smooths the age pattern of MLC1.

For PMD-unadj, theoretically there should not be any jumps in m_x because the age-specific death rates between the last estimation period and the last projection period is assumed to change linearly over the projection period. Practically, PMD-unadj can produce jumps in m_x when the life expectancy is not derived from projected age-specific mortality rates, but is pre-projected as in the *World Population Prospects* (United Nations, 2017a). However, the rates of sex crossovers produced by PMD-unadj can be substantial if males and females have had different trajectories of mortality decline in the past and a sex-specific age pattern of mortality improvement in the future years is applied. Such sex crossovers produced by PMD-unadj are more likely to occur when the female-male gap in life expectancy is narrow. In such cases, the PMD with consistent application of sex ratio adjustments for age-specific death rates can largely reduce the occurrence of sex crossovers (tables 1 and 2).

Because the subset of countries with high quality data in the Human Mortality Database (HMDa) witnessed a decline in mortality with very limited fluctuation in the past several decades (Ouellette, Barbieri, and Wilmoth, 2014), these countries are likely to have fewer sex crossovers and jumps in m_x than the other two groups of countries when the MLC method is applied. On the other hand, HMD countries with lower quality data (HMDb) have had greater fluctuations in mortality rates over time, leading to higher rates of sex crossovers and jumps in m_x than in HMDa countries and non-HMD countries with lower quality data (Leon, 2011; Meslé, 2004).

Overall, the MLC method, regardless of its variants, works quite well for HMDa countries in terms of relatively fewer sex crossovers and jumps in m_x. The PMD-unadj method performs better than all three MLC variants in the case of jumps in m_x for all country groups, particularly for females. The relatively higher rate of sex crossovers in m_x produced by PMD-unadj can be avoided or largely reduced when an appropriate adjustment for sex ratio of m_x is applied, as was done for many countries in the *2017 Revision* (table 1). Mainly for these reasons, PMD is the preferred method for HMDb and non-HMD countries in the

2017 Revision. However, the adjustment of the sex ratio of m_x should be done in a systematic and consistent way in the next revision.

The results of the study suggest that additional avenues could be investigated to further refine the projections of age-specific mortality rates. First, an application of a sex-coherent age pattern of mortality improvement for PMD, rather than an ad-hoc adjustment of the sex ratios, may further reduce or avoid the sex crossovers in the projection periods. Second, for the MLC method, the routines can be extended to test the averaging of the mortality age pattern over a flexible number of periods, instead of just the whole estimation period, as well as testing the smoothing of the averaged age patterns. Third, in generating base-year mortality patterns, especially for HMDb countries and non-HMD countries, it may also be useful to incorporate regional or subregional patterns instead of merely country-specific patterns. Finally, refining the female-male gap in e_0 in BHM projection may also be helpful to avoid sex gaps in e_0 that are too narrow in the projection period, which may be a contributing factor to sex crossovers in m_x in several countries.

In sum, this study has identified some weaknesses and strength in both the MLC and PMD methods for projecting mortality patterns of countries with varying degrees of data quality in the *World Population Prospects*, providing useful guidance for the selection of appropriate methods. It also suggests some methodological improvements that should be applied in using PMD in the next revision of the *World Population Prospects,* planned for 2019.

Appendices

TABLE A1. LIST OF HMD AND NON-HMD COUNTRIES BY PROJECTION METHOD USED IN THE *WORLD POPULATION PROSPECTS: THE 2017 REVISION*

Group of countries	Method	No. of countries	List of countries
HMDa		25	
	MLC1	8	Belgium, Denmark, France, Japan, Slovenia, Spain, Switzerland, United States of America.
	MLC2	5	Australia, Canada, Luxembourg, New Zealand, United Kingdom
	MLC3	12	Austria, Finland, Germany, Greece, Iceland, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Sweden
HMDb	PMD	13	Belarus, Bulgaria, Chile, Taiwan Province (China), Czechia, Estonia, Hungary, Latvia, Lithuania, Poland, Russian Federation, Slovakia, Ukraine
Non-HMD	PMD	117	Albania, Algeria, Antigua and Barbuda, Argentina, Armenia, Azerbaijan, Bahamas, Bahrain, Bangladesh, Barbados, Belize, Benin, Bhutan, Bolivia (Plurinational State of), Bosnia and Herzegovina, Brazil, Brunei Darussalam, Burkina Faso, Burundi, Côte d'Ivoire, Cambodia, Chad, China, China, Hong Kong SAR China, Macao SAR China, Colombia, Costa Rica, Croatia, Cuba, Curaçao, Cyprus, Dem. People's Republic of Korea, Djibouti, Dominican Republic, Ecuador, Egypt, El Salvador, Eritrea, Fiji, French Polynesia, Gambia, Georgia, Ghana, Grenada, Guadeloupe, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, India, Indonesia, Iran (Islamic Republic of), Iraq, Jamaica, Jordan, Kazakhstan, Kuwait, Kyrgyzstan, Liberia, Libya, Madagascar, Malaysia, Maldives, Mali, Malta, Martinique, Mauritania, Mauritius, Mayetta, Mexico, Mongolia, Montenegro, Morocco, Nicaragua, Niger, Nigeria, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Puerto Rico, Qatar, Republic of Korea, Republic of Moldova, Romania, Réunion, Saint Lucia, Saint Vincent and the Grenadines, Saudi Arabia, Senegal, Serbia, Seychelles, Sierra Leone, Singapore, South Sudan, Sri Lanka, Suriname, TFYR Macedonia, Tajikistan, Thailand, Timor-Leste, Togo, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, United Arab Emirates, Uruguay, Uzbekistan, Venezuela (Bolivarian Republic of), Viet Nam, Yemen

TABLE A2. MULTILEVEL MULTIVARIABLE LOGISTIC ANALYSIS (ODDS RATIOS) OF HAVING A SEX CROSSOVER IN m_x

		Methods			
	Total	LCM1	LCM2	LCM3	PMD
LCM1 (reference)	1.00	--	--	--	--
LMC2	0.80***	--	--	--	--
LCM3	0.29***	--	--	--	--
PMD-unadj	2.28***	--	--	--	--
Ages 0 (reference)	1.00	1.00	1.00	1.00	1.00
Ages 1-4	1.67***	1.71***	1.02	4.31***	2.51***
Ages 5-14	0.35***	0.55***	0.34***	0.94	0.14***
Ages 15-69	0.03***	0.03***	0.02***	0.16***	0.01***
Ages 70-89	0.01***	0.02***	0.02***	0.05***	0.00***
Ages 90+	0.11***	0.23***	0.22***	0.20***	0.01***
Period 2015-2020 (reference)	1.00	1.00	1.00	1.00	1.00
Period 2020-2030	0.94	1.03	0.94	0.69	0.99
Period 2030-2050	1.06	1.22	1.12	0.45***	1.36*
Period 2050-2075	1.08	1.07	1.07	0.36***	1.78***
Period 2075-2100	0.90	0.73*	0.71*	0.25***	2.04***
e_0 dif<2 years	18.8***	13.1***	14.4***	59.6***	54.1***
$2 \le e_0$ dif <3 years	4.82***	4.09***	4.52***	9.57***	7.08***
$3 \le e_0$ dif <4 years (reference)	1.00	1.00	1.00	1.00	1.00
$4 \le e_0$ dif <5 years	0.48***	0.51***	0.52***	0.25***	0.43***
e_0 dif ≥ 5 years	0.14***	0.12***	0.11***	0.01***	0.14***
HMDa countries (25)	0.61	0.62	0.33	0.50	0.98
HMDb countries (13)	17.6***	62.5***	78.2***	2.22	18.5***
Non-HMD countries (117) (reference)	1.00	1.00	1.00	1.00	1.00
degree of freedom	18	15	15	15	15
rho	0.38	0.60	0.68	0.68	0.47
-log likelihood	23149.5***	5729.0***	6824.0***	2418.7***	7031.8***

NOTE: (1) The results for each of four methods are based on 57,970 observations (= 22 age groups * 17 five-year periods * 155 countries) with the dependent variable being a dichotomous variable of having a sex crossover in m_x or not. All models adjust the intra-country correlation of observations (rho). The total sample includes all observations (57,970*4=231,880) in these four projection methods. (2) e_0 dif: the female-male gap in life expectancy (e_0^f - e_0^m). (3) Numbers in the parentheses refers to number of countries in a given group. (4) --: the factor is not included in the model. (5) *p<0.05, **p<0.01, ***p<0.001.

TABLE A3. MULTILEVEL MULTIVARIABLE LOGISTIC ANALYSIS (ODDS RATIOS) OF HAVING A JUMP IN m_x

	Both sexes	**Males**	**Females**
LCM1 (reference)	1.00	1.00	1.00
LMC2	1.84***	1.81***	1.91***
LCM3	5.58***	4.55***	7.19***
PMD-unadj	0.78***	1.28***	0.26***
Ages 0 (reference)	1.00	1.00	1.00
Ages 1-4	2.18***	2.16***	2.30***
Ages 5-14	0.86**	0.83*	0.88
Ages 15-69	0.17***	0.14***	0.20***
Ages 70-89	0.08***	0.09***	0.07***
Ages 90+	0.54***	0.84*	0.27***
Period 2015-2020 (reference)	1.00	1.00	1.00
Period 2020-2030	0.02***	0.03***	0.01**
Period 2030-2050	0.01***	0.02***	0.01***
Period 2050-2075	0.02***	0.02***	0.03***
Period 2075-2100	0.04***	0.04***	0.10***
$e_0^m < 70$ years	0.17***	0.22***	--
$70 \leq e_0^m < 75$ years	0.48***	0.53***	--
$75 \leq e_0^m < 80$ years (reference)	1.00	1.00	--
$80 \leq e_0^m < 85$ years	1.69***	1.76***	--
$85 \leq e_0^m < 90$ years	1.80***	2.23***	--
$e_0^m \geq 90$ years	0.35	0.28	--
$e_0^f < 70$ years	1.54**	--	1.67**
$70 \leq e_0^f < 75$ years	1.50***	--	1.23
$75 \leq e_0^f < 80$ years (reference)	1.00	--	1.00
$80 \leq e_0^f < 85$ years	1.51***	--	5.18***
$85 \leq e_0^f < 90$ years	1.43**	--	3.39***
$e_0^f \geq 90$ years	0.76	--	0.92
HMDa countries (25)	0.18***	0.20***	0.28***
HMDb countries (13)	2.68***	3.04***	2.69***
Non-HMD countries (117) (reference)	1.00	1.00	1.00
Males	1.00	--	--
Females	0.93**	--	--
degree of freedom	25	19	19
rho	0.26	0.27	0.29
-log likelihood	20620.1***	10932.3***	9274.3***

NOTE: The results of males and females are based on 231,880 observations (= 22 age groups * 17 five-year periods * 155 countries * 4 projection methods) with the dependent variable being a dichotomous variable of having a jump in m_x or not between two adjacent periods. The results for both sexes combined are based on 463,760 observations. All models adjust the intra-country correlation of observations (rho). (2) Numbers in the parentheses refers to number of countries in a given group. (3) --: the factor is not included in the model. (4) *p<0.05, **p<0.01, ***p<0.001.

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